Introduction

There is presently a search for new, alternative sources of energy. One of these sources is wind power, which is currently generated primarily from large rural wind farms of horizontal axis wind turbines (HAWTs). Unfortunately, wind farms tend to be far from the urban areas where most of the power is being used, resulting in large transmission costs and electrical losses. One logical solution to this problem would be to erect wind turbines in cities, but there are many obstacles to doing this. The biggest is that "urban" wind is generally very turbulent, while the HAWT—which is widely considered the best type of wind turbine—cannot operate in turbulent wind, rather, requiring clean, steady wind. Certain types of vertical axis wind turbine (VAWT), on the other hand, can operate in turbulent winds.

A horizontal axis wind turbine (HAWT) has a vertical tower with a horizontal axis. The blades are straight and attached to the end of the axle. On a HAWT, wind hits the blades, usually turned at an angle, and pushes them out of the way. Because the blades are angled, the wind in turn rotates the hub and turns the generator. A vertical axis wind turbine (VAWT) has a vertical axle which also acts as the tower. There are many different types of VAWTs, but they all have the gears, generator, and electronics at the base of the tower which eliminates the need to have wires going to the top. The major advantage is that the wind can come from any direction and the turbine does not need to be pointed into it.
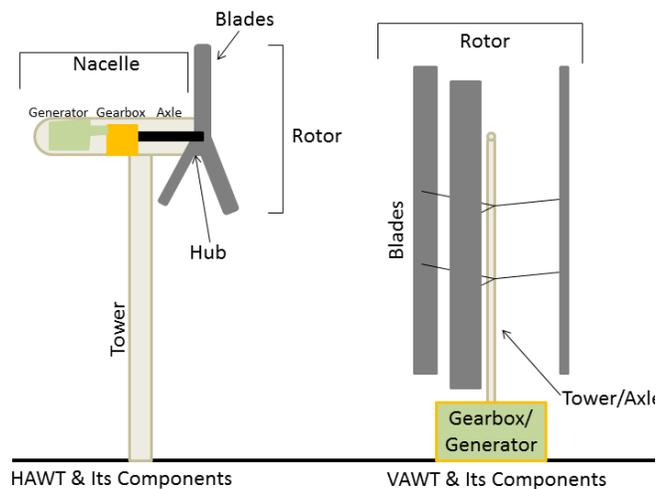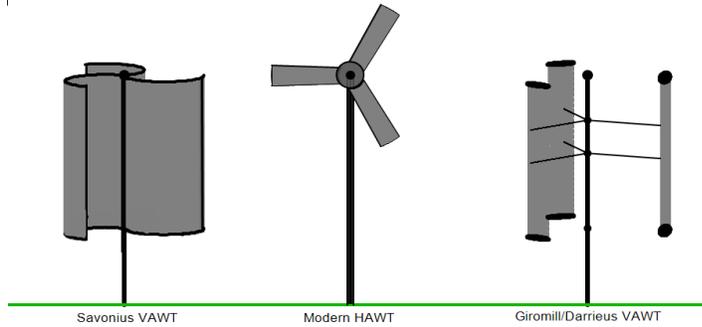

Fig. 1: General differences between horizontal axis wind turbines (HAWTs) and vertical axis wind turbines (VAWTs) with parts labeled. (Image by Author)

Fig 2: The three basic types of wind turbine (image by author)

Savonius VAWT   Modern HAWT   Giromill/Darrieus VAWT

There are two prominent types of VAWT, Savonius and Darrieus, both originating in the early 20th century. Figure 2 shows these VAWT designs, along with a HAWT for visual comparison. The two designs are complete opposites in fundamental blade operation principle but identical in basic operation and mechanics. The basic Savonius design is drag based—the wind essentially pushes on the blades. The Darrieus design, on the other hand is lift based—the wind essentially pulls the blades because of pressure differentials. Additionally, both of these designs can be twisted about the axle, giving many advantages to the turbine's operation but being much more complicated to construct.

Despite being relatively inexpensive and requiring little maintenance, Savonius turbines are not often used because they are inefficient in comparison with HAWTs and Darrieus turbines. Savonius designs operate similarly to cup anemometers[1] which are also drag-based. Less drag occurs when the wind is pushing against the scoops than when the scoops are moving with it; this differential causes torque and rotation on the shaft (Savonius wind turbine, 2009). The turbines will also only spin as fast as the wind is blowing because they are drag based; thus, their tip speed ratio is less than or equal to one. Savonius turbines, therefore, have slow rotations per minute (RPM) ratios and high torque creating the need to be geared up[2] when
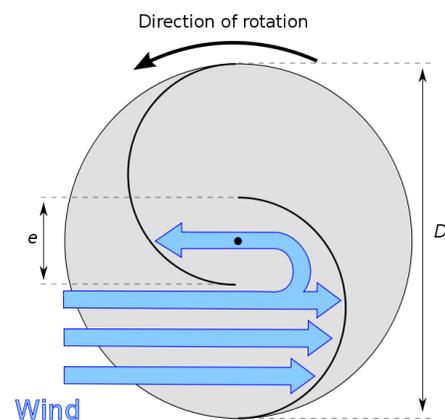

Direction of rotation

Wind

Fig. 3: Diagram of wind patterns within a two bladed Savonius turbine. (Savonius wind turbine, 2009)

---

[1] A cup anemometer is a device that measures wind speed using several cups that are blown by the wind, spinning around a pole/axle that is vertical.
[2] To gear up the rotation of an axle, a large gear is placed on the first axle and a small gear is placed on the second axle; the cogs line up and cause the second axle to spin much faster.

used for power generation. This requires more RPM to provide more wattage (unit of measurement of electrical power output). Fortunately, Savonius turbines capitalize on turbulent winds like those found near buildings, creating higher efficiency on buildings. Because of these factors, Savonius designs are rarely used industrially but can be successful when independently constructed to provide power on a smaller scale (Savonius wind turbines—Wind, 2008).

Savonius turbines are better suited than HAWTs for rooftop mounting for several reasons. They don't require an inefficient tower (apart from the tower that also serves as the axle), and HAWTs must be kept pointing either into or with the wind to operate, unlike VAWTs. HAWTs also experience damaging vibrations, which have harmful effects on both the turbines themselves (including any structure they're mounted on) and the people living or working near the turbines, but VAWTs don't have these vibrations. The vibrations primarily cause noise and pulsations and are created because of turbulence that occurs as a HAWT's blades spin past the tower. The turbulence of the airflow is caused by the blades moving perpendicularly to the wind, and further variation in airflow causes additional vibrations each time the blades pass the tower. The vibrations occur at a low enough frequency that they are often inaudible to humans, but they can also cause certain parts of buildings, such as windows, to vibrate (Van Den Berg, 2004).

The American Wind Energy Association (AWEA) disapproves of wind turbines mounted directly on rooftops, insisting that a tower should be utilized on top of the roof to bring the turbine above all of the turbulence of an urban environment. Instead, one could save on the cost and potential dangers of a long tower (proposed by the AWEA to be 80+ feet long) by designing a turbine to capture those turbulent winds. If turbines are built into existing towers, like lampposts or telephone poles, then using a tower may be worthwhile; otherwise, it makes more sense to utilize turbulence-tolerant designs on rooftops, which actually create their own

windflow patterns that can channel the wind into the turbine if placed correctly.

In previous qualitative studies on a scale model of a real building, fans were used to simulate a randomized wind flow pattern around the building, in order to explore rooftop placement options. The wind was designed to be as turbulent as possible to simulate the flow of real winds around urban buildings. I found that the wind hitting the flat side of a building created a concentrated channel of wind ideal for turbines placed directly on the leading edge of the building. However, when the wind hit one of the building's corners, the flow created a "dead" spot where there was almost *no* airflow at the leading corner of the roof, but a very *strong* flow along the edges of the roof further away from the corner. Therefore, we know that buildings do channel airflow where turbulent-tolerant turbines could take advantage of it.

Quantitative analysis of the best design for VAWTs on rooftops was also conducted using scale model testing. I found that an H-Darrieus VAWT (with two vertical airfoils) was more effective than a simple Savonius VAWT (with two vertical half-cylinders), but both designs were inconsistent in turbulent winds and couldn't always self-start. A third turbine tested was a Savonius turbine with a helical twist applied about the axle. This "Twisted Savonius" design, originating only within the past fifteen years, was very consistent in operation and also had much higher average power output in watts than the simple designs. I concluded, therefore, that the best type of wind turbine in turbulent rooftop winds is the Twisted Savonius. Because the shape of the Twisted Savonius design is very complex, the process of constructing a Twisted Savonius wind turbine is also complex, requires expensive materials and machinery to build, and carries a high cost. In order to simplify the method of this design's construction, its shape must be better understood.

Little research has been done on Twisted Savonius wind turbines, and it is unclear who originally conceived the idea to twist the blades of a Savonius wind turbine; relevant patents state no specifics about the general design (instead focusing on specifics to their models). If a twist angle of pi radians (180°) is achieved for a two bladed turbine, the wind can come from any angle simultaneously and provide a constant pushing force on the turbine's blades. Additionally, because the design is drag-based it doesn't matter if different parts of the blade(s) receive different forces (from the turbulent winds), therefore the Twisted Savonius design should be better than a Twisted Darrieus design operating in a turbulent rooftop environment. In addition to these theoretical principles, a small amount of research has been done on the Twisted Savonius design. Saha and Rajkumar (2005) investigated different twist angles for a three bladed Savonius VAWT through low speed wind tunnel testing. Forces on six small VAWT models, each with identical parameters (including weight), were observed, including torque and angular velocity (rpm), the primary dependent variable. A "power curve" of each of the designs was created from the results and showed that more twist angle didn't necessarily mean a higher RPM; this would contradict theoretical principles that more twist is better, but their methods of not adding additional material to compensate for the twist likely skewed their results. They did verify that the twisted blades had a much more constant torque than the straight ones, meaning that they should require less maintenance and have a longer life-span.

Another study on the Twisted Savonius design by Hussain, Mehdi, and Reddy (2006), attempted to increase the efficiency of Savonius VAWTs, by twisting the blades. This experiment utilized Computational Fluid Dynamics (CFD), a computerized, instead of a physical, model. After configuring the software to place a simulated wind-flow on the designs, the simulation was carried out and performance data was collected for designs with twist angles

at intervals of 5° from 0 through 60. It is clear from their results that increasing the twist

increases the efficiency of the model, until the twist reaches 45^0 at which point it begins to

decline. With the maximum efficiency at 45^0 of twist, it was found that there was also the

greatest positive surface area when the twist equaled 45^0. The surface area was calculated by the

concave area of one blade minus the convex area of the other blade, so that the forces would

produce a net force on the concave blade, spinning the turbine.

These studies clearly show that increasing the twist angle of a Savonius wind turbine's

blades increases its efficiency, at least to a point. Theoretically, the greater the twist angle, the

more efficient the turbine will be until the angle of twist reaches 180^0, at which point the blades

of the turbine reach all the way around the axle of the turbine. So why does the efficiency decline

from 45^0 on? In order to understand the twist, a crucial piece of the Twisted Savonius design,

and determine a better method of building it, it is critical to get to the root of the blades' shape,

which previous studies have approached inconsistently. Scale model testing and computerized

airflow modeling have been done to explore the different aspects of optimizing the twisted

Savonius design, but it is necessary to explore the *geometry* of the blade shape to understand

why there is an ideal value of twist. Exploring this value, thought to be at 45^0, could provide

crucial information about the geometry of the shape that suggests why it works so well fluid-

dynamically. It could also explain why more twist reduces efficiency after 45^0 of twist. Since the

goal is to model the design geometrically and be able to easily adjust the parameters of its

geometric structure, the use of a symbolic geometry program is ideal.

Symbolic geometry programs can draw accurate geometric drawings and use numbers,

variables and/or equations to constrain the drawings. These constraints involve many variables

which can be adjusted or even animated within the program. In addition to creating these

extremely accurate and mathematically correct drawings, symbolic geometry programs can make calculations of lengths, angles, and more within a drawing, both numerically and symbolically in terms of the variables used (Oke, 2010). This project utilizes the symbolic geometry program *Geometry Expressions*.

The objective of this project is to explore the geometric shape of the twisted Savonius VAWT in order to optimize the design and develop a simpler method of constructing it. While investigating the shape, a particular focus was on the differences in shape as the angle of twist is varied. I predicted that there is something causing a reduction of surface area within the geometry of the blade that causes it to lose efficiency at higher levels of twist. Calculations on different parts of the model were also used to learn more about the shape including an attempt to find the equation for the shape's surface area in terms of the turbine's radius, the twist angle and other parameters. I was also able to determine the three-dimensional parametric equation for the surface of the blade. After the geometry of the blade was studied, methods for "unrolling" the blade into a flat shape that could be stretched over a frame to build the turbine were studied. If successfully implemented, this method would drastically reduce the cost of the turbines, making widespread use much more feasible and allowing generation of wind energy at the location of power usage to become a regular practice.

Methods

Procedures: Both a top view and a side view of the wind turbine were made. The top view was used primarily to make calculations on the turbine, but has fewer direct applications. The mathematical process of creating the side view model is included below, and includes mathematical instructions but not Geometry Expressions keystrokes. This method—which uses an elliptical model to replicate tilted circles in a pseudo-three-dimensional plane within a two-

dimensional plane—was originally suggested by my mentor, the creator of Symbolic

Geometry/Geometry Expressions, and I developed the actual method following that principle.

The twist of the turbine is constrained as theta, the rotation of the turbine is X, and s and t

represent how far up and around the blade an arbitrary pair of points are. When the X variable—

which controls the spin—is animated, a visual of the wind turbine operating in the wind as it

would look from the side is produced. We can "fill in" the entire blade by constructing traces,

which are essentially the path along which a certain line, curve, locus, etc. moves traced out a

given number of times through a certain interval (the path of an object as a parameter (usually a

proportional point along a curve) changes). The ability of *Geometry Expressions* to create semi-

transparent surfaces out of traces allows both blades to be seen as they are spinning which is

especially useful for trying to study and understand the shape. This project assumes that the

Savonius wind turbine's blades have a semicircular cross section (horizontally). This is often the

case with Savonius turbines but some have cross sections that are more elliptical. The entire

construction process of the side view model is described below.

Three ellipses are drawn first. The equation of each is constrained so that two ellipses are

always tangential to each other and the third ellipse, and the first two ellipses rotate within the

third as X is varied. The following equations are used to constrain the ellipses, with all of the

letters and symbols being used as ellipse parameters except for X which controls spin.

$$\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-k}{b}\right)^2 = 4; \quad \left(\frac{x-h-a*\cos(-X)}{a}\right)^2 + \left(\frac{y-k-b*\sin(-X)}{b}\right)^2 = 1;$$

$$\left(\frac{x-h-a*\cos(\pi-X)}{a}\right)^2 + \left(\frac{y-k-b*\sin(\pi-X)}{b}\right)^2 = 1$$

Next, points are created on the smaller ellipses as they were in the top view and with exactly the

same constraints. When all six points have been drawn and constrained, the entire drawing is

copied and pasted (the second set will not yet be visible as it is constrained to be on top of the first). One of each ellipse equation is revised with a "-ht" in the "y" section. When all three ellipses are duplicated below the original ones at a distance of ht, theta is added to every constraint right next to the X (except in the outer ellipse equation) as in this equation:

$\left(\frac{x-h-a*\cos(\pi-X+\theta)}{a}\right)^2 + \left(\frac{y-k-b*\sin(\pi-X+\theta)}{b}\right)^2 = 1$. As before, line segments are drawn from the top set's t points to those on the bottom. Then a point is placed on each line segment and constrained to be proportional at s. The loci of these points are constructed in terms of t from zero to pi, and then the trace of each locus is constructed in terms of s, from zero to one, and the trace of each line segment is constructed in terms of t, from zero to pi. All of the constraints are hidden, giving the final figure (4, right), which can be animated to operate or twist.
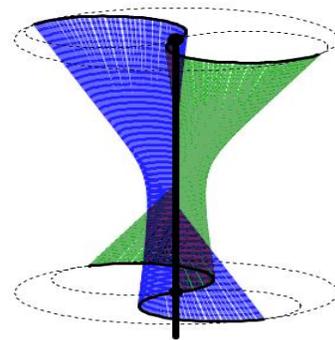

Figure 4: The basic side view "3D" model twisted (theta) about 120º and rotated (X) about 30º.

Geometric constraints require that multiple sections of blade are necessary in a turbine for it to be twisted the full 180º, which necessitates a slightly extended procedure for construction, see the results for the reasons for this added process. Four sections identical to the one already described are twisted 45º or pi radians, and stacked to form this more effective design. To create the visual model of the full, stacked turbine, five sets of rotating (with X) ellipses were created instead of two. The variables ht, theta and X were also constrained as before, but using fractions so the model's sections would be positioned, twisted and rotated evenly between each section. Though the construction process is very long and tedious, the full side view model, including all five elliptical sections, can be built in about 45 minutes using a refined keystroke method. The result allows one to conceptualize the turbine's shape, especially when it is animated, an easy possibility thanks to the methodical use of the variables X and theta.

Figure 5: The full side view model
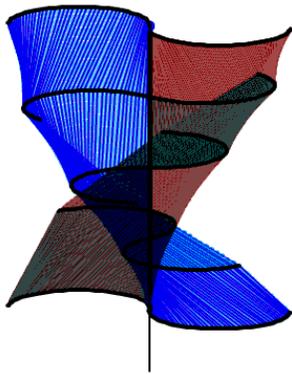
In addition to building and observing a computerized model of the twisted Savonius wind turbine, I developed a method which allows one to create the turbine from a flat piece of material without specialized equipment. The shape of the twisted Savonius blade cannot be unrolled into a flat surface, as with a sphere, because of the relations in its three-dimensional cross-sectional planes. Instead, I used triangles based on control points and line segments taken from the top view model, and then incorporated the vertical dimension, with the Pythagorean Theorem, to create a flat surface with inner fold lines that can be erected into the blade shape. A brief explanation of the process of doing this using 4 sections to approximate the turbine follows, but this method was also expanded to build approximations of 6, 8, 10, 12, 14, 16, 18, 20, 24, 28 and 32 sides.

In the cutout approximations, points were constrained around the first half of each circle in the top view at even intervals. The second (left) circle was constrained identically but with theta added to all of the points. Next, line segments were drawn between the corresponding points (1 on circle 1 to 1 on circle 2, etc.) and on diagonals between them (2 on the first circle to 1 on the second, etc.).

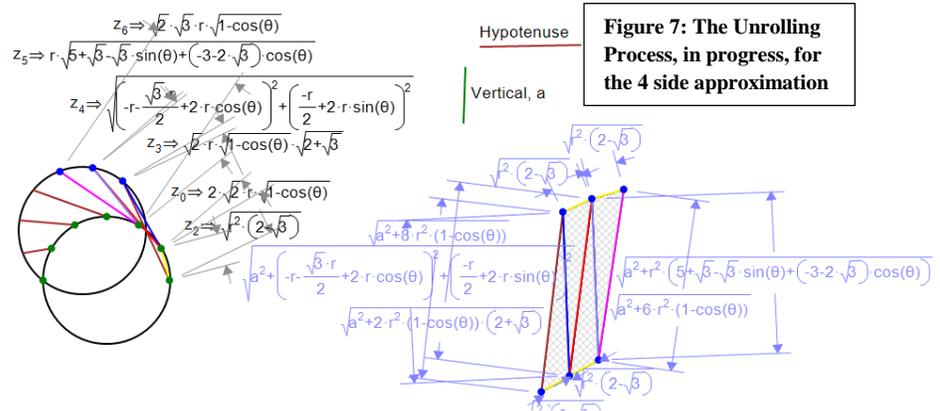**Figure 6: Full frame for line calculations in the top view**
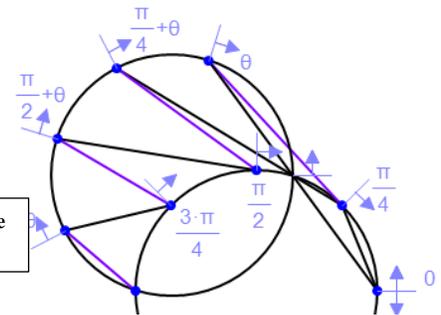
Once the circles were completed, the symbolic lengths of every line segment were calculated. Each of these equations represents the adjacent side of a right triangle, which appears as a line in the top view. The variable 'a' was used to represent the vertical, opposite side which appeared as a point in the top view. The remaining triangle side, the hypotenuse, was determined using the Pythagorean Theorem: the calculations (representing the b side) must be squared, a^2 must be added, and the square root of the total must be taken to find the constraint, representing the c side.

One side of each unrolled triangle doesn't need to have a incorporated into it: the 4 sides that sit at the top and bottom of each blade. These constraints are copied directly from the calculation of the symbolic distance between any two consecutive points on the same circle.

Figure 7 shows the progression of the unrolled figure as more and more of the triangles are added with color coding showing their relations to the circles. The first "vertical" side of the first triangle, left to right, uses the first black line's equation on the circles: from point 1 to point 1. The short bottom side uses the unchanged distance between two consecutive points on the same circle. The last

Figure 7: The Unrolling Process, in progress, for the 4 side approximation

$$z_6 \Rightarrow \sqrt{2} \cdot \sqrt{3} \cdot r \cdot \sqrt{1-\cos(\theta)}$$
$$z_5 \Rightarrow r \cdot \sqrt{5 + \sqrt{3} - \sqrt{3} \cdot \sin(\theta) + \left(-3 - 2\sqrt{3}\right) \cdot \cos(\theta)}$$
$$z_4 \Rightarrow \sqrt{\left(-r - \frac{\sqrt{3}}{2} + 2 \cdot r \cdot \cos(\theta)\right)^2 + \left(\frac{-r}{2} + 2 \cdot r \cdot \sin(\theta)\right)^2}$$
$$z_3 \Rightarrow \sqrt{2} \cdot r \cdot \sqrt{1-\cos(\theta)} \cdot \sqrt{2 + \sqrt{3}}$$
$$z_0 \Rightarrow 2 \cdot \sqrt{2} \cdot r \cdot \sqrt{1-\cos(\theta)}$$
$$z_2 \Rightarrow \sqrt{r^2 \cdot \left(2 - \sqrt{3}\right)}$$

side of the triangle (also the first side of the next) is the first colored line, from point 2 to point 1. The constraint process continues until all of the lines on the circles have been used, and the last vertical side, representing a point on the top view, is simply a, because the adjacent side is equal to 0. After all of the constraints, point labels, etc. are hidden, the unrolled figure is reflected about the last side, constrained as a, which is also constrained to be oriented vertically. Appropriate parameters are pi/4 for theta, 1.83 for a and 1 for r, the a:r ration the same as the one used by previous researchers (Saha, 2005). At times the aspect ratio was rounded to 2:1 for simplicity purposes. Creating a polygon with all of the outermost side lengths allows for a real calculation of the shape's area. Unfortunately a symbolic output, which would be extremely useful for analyzing shape changes in terms of height, radius and theta changes, does not seem to exist in a finite form, because *Geometry Expressions* cannot complete the calculation after 5 minutes (it usually takes milliseconds) and the partial equation it gives is immensely long. The completed 4 sided approximation is in Figure 8.

Figure 8: Completed 4 sided approximation.
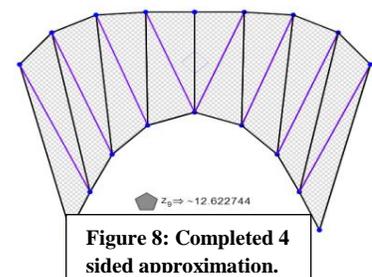
The figures and animations produced allow observations to be made about how the turbine's shape functions geometrically. This included mathematical and qualitative investigations of surface area and surface distribution through the blades. Several calculations were made on the figures and then manipulated in *Maple* (CAS) to produce more graphs and actual three-dimensional images as well as to make algebraic and calculus calculations. Two-dimensional graphs were created in *Geometry Expressions* after manipulation in *Maple* as well.

Data Analysis: No strictly quantitative data were collected, so there was no data analysis in the formal sense. The formulas output from *Geometry Expressions* were often manipulated in *Geometry Expressions* and *Maple* as described in the procedures. Graphs were created to analyze how close the approximations were to the limit of the surface area for n=infinity triangles, utilizing the basic calculus principles of limits and derivatives. In theory, the calculus of integrals can be used to find the exact surface area of the actual blade—not just the triangle approximation—however, this process requires advanced calculus principles. The data points for each triangle approximation were manually analyzed and interpreted, much like traditional data collection, so quantitative data analysis was conducted for the surface area investigation.

Results

The Squeeze: As previously mentioned, the Savonius wind turbine's blade is essentially squeezed as it is twisted without any supports between the twist points, which was discovered when the first animation of twisting the turbine was viewed. To model the squeeze, it was first looked at from the simplest view: the top. The angle between the circles in the top view was constrained as theta and there are proportional points at t and theta+t on the circles with a line segment between them. By moving the value of s, a proportional point on the line segment, the green locus moves up and down through all of the turbine's horizontal cross sections (fig 21b,

below). We can see in Figure 9a that the

locus of that point (in green) in terms of t is

always a circle but gets larger and smaller:

Fig 9 a & b: Traces of the loci (green) show the cross-sectional radius' squeeze in the top and side views

One question that comes out of this

is at what point is the squeeze the most

extreme (or where is it most squeezed) in terms of s? This happens when the distance between

the two intersection points of the circle is equal to the diameter of the locus. This is because the

locus always goes through two points, the intersections of the circles. The point of most extreme

squeeze is where the locus circle intersection points are directly opposite each other on the locus.

This value for s is found by calculating the locus' symbolic equation, drawing a circle and

constraining it to have the same equation as the locus, then calculating the circle's radius and the

distance between the circle intersections. When the radius is half the distance, the correct value

for s has been found. Fortunately, the minimum radius is found at 0.5s, or halfway between the

top and the bottom. This means that the squeeze is symmetrical. Now the question is what effect

does theta (the twist) have on the amount of squeeze? Figure 10

illustrates how the radius, constrained at the maximum squeeze

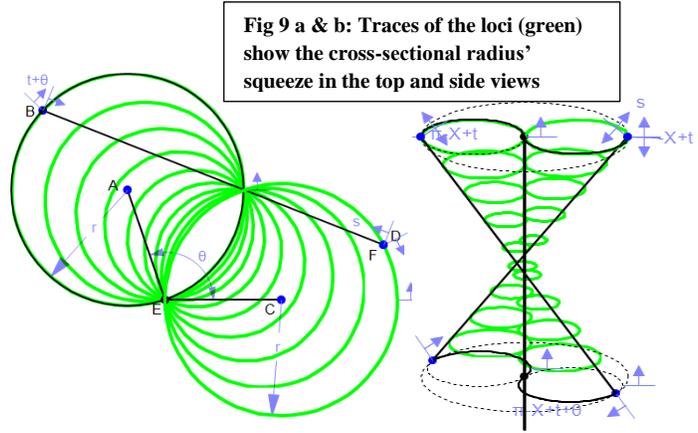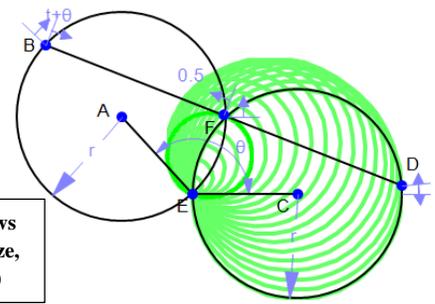value of 0.5 affects the amount of squeeze, the smaller the locus,

the more the squeeze.

Fig 10: the trace of the locus (green) shows the effect of adjusting theta on the squeeze, when it is at its most extreme point (s=.5)
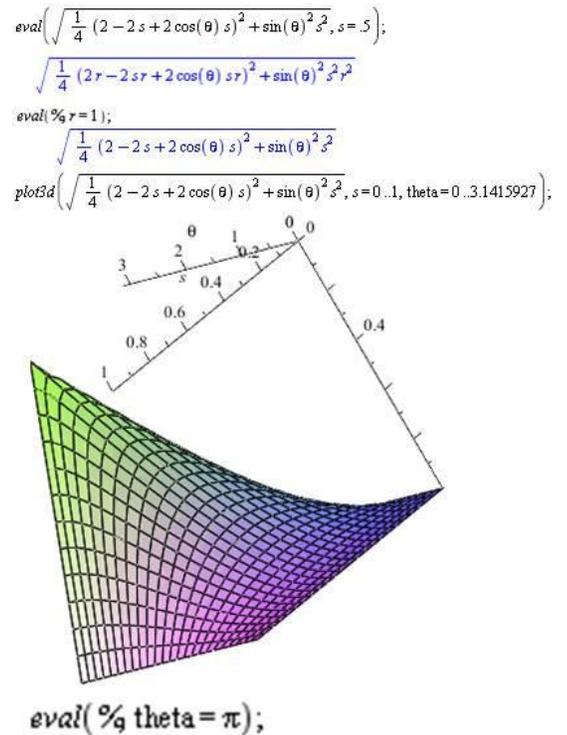
The squeeze can also be modeled in a computer algebra system (CAS), such as *Maple*.

To do this, the *Geometry Expressions* calculation for the symbolic radius of the locus of the point

constrained at s is copied and pasted into *Maple*. Then, the constant r is set at 1 and theta is

replaced with pi. Plotting the result gives the amount of squeeze moving up/down the turbine

when it is twisted pi radians (*Maple* works almost exclusively in radians). Going back to the
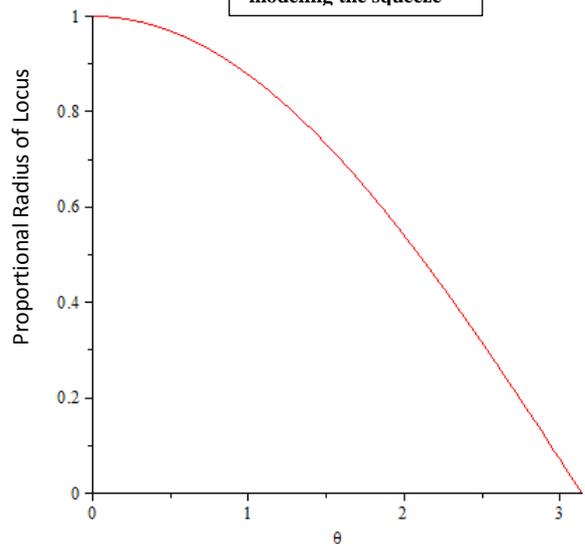
theta equation, a three-dimensional plot of the squeeze can also be created. In this plot, the

unmarked axis is the radius of the locus and the other two dimensions represent theta and s.

Finally, there is a two-dimensional plot of how the twist affects the radius at the minimum radius

and a calculation of the radius at a specific theta value. The relevant pieces of the Maple

worksheet are right in Figure 11.

The last graph (Figure 12) is most useful because it

shows how much compromise in squeeze there is for a

given angle of twist on the turbine. Avoiding this issue is

the ideal solution, and with the goal being to make the

turbine as geometrically simple as possible, it was

determined that stacking four sections of turbine, each

rotated 45° was the ideal solution for these conditions.

This way there is a minimal amount of squeeze (45°=pi/4

radians, ~.7 radians for graphical reference), but also a

minimal number of sections to separately construct and stack.

Figures 11 & 12: *Maple* (CAS) Worksheet modeling the squeeze

The Surface Area Limit: As expected, each

approximation model had an area that was slightly

bigger than the previous, and the difference between

each area grew smaller and smaller as the number of

sides, n, got bigger. The slope of the line segment

connecting the twenty-eight and thirty-two side

approximations was only .001349, with the rise being

about .0054 and the run being the change in n, 4.

Calculating all of the other line segments on the graph, using *Geometry Expressions* as the

grapher by constraining points and showing the axes, they generally got smaller and smaller,

confirming the visual evidence of the actual graph numerically. The main concepts of calculus,

limits, derivatives and integrals are very relevant to this piece. The goal is to find the limit of the

surface area as n approaches infinity, which represents a definite integral on an unknown (and

possibly non-existent) function. The derivative is used to find the rate of change (slope) between

each approximation's data points. Figure 13, below, is the graph in *Geometry Expressions* with
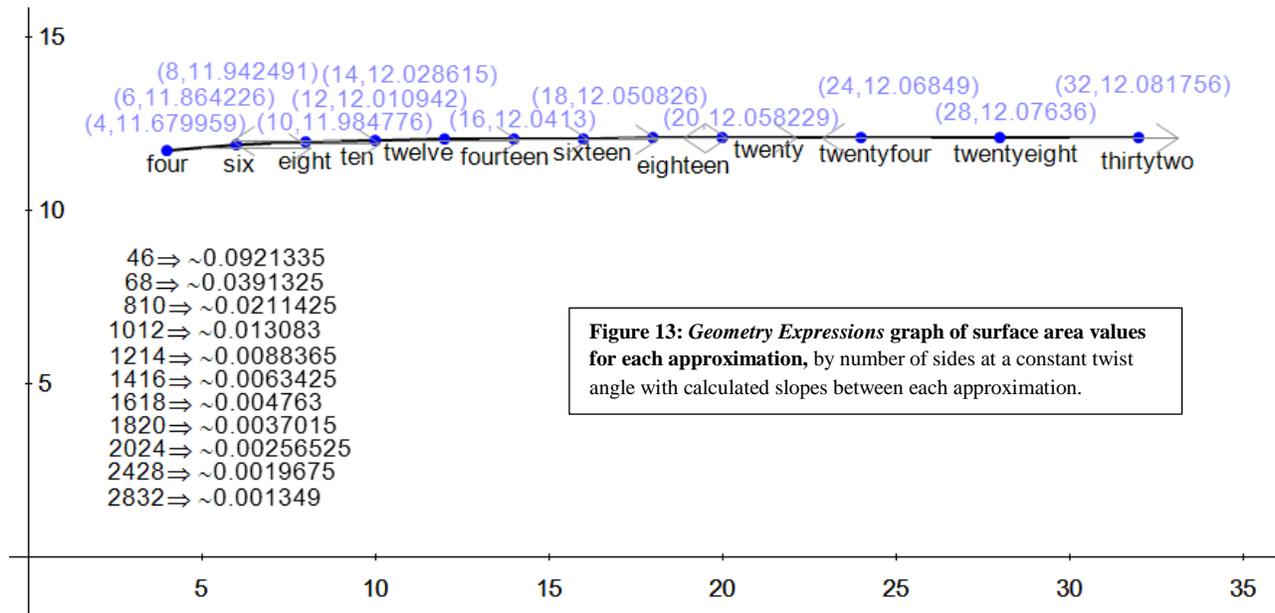
the data points and the slopes between each approximation.

Figure 13: *Geometry Expressions* **graph of surface area values for each approximation,** by number of sides at a constant twist angle with calculated slopes between each approximation.

In order to show this limit with all four mathematical methods: geometrically (building

models), numerically (areas and rates of change between areas), graphically (with the graph of

the area compared to n), and algebraically; it is useful to try to find an approximate equation to

represent this limit to give an algebraic representation. Because of the almost logarithmic shape

of a finite limit-approaching graph, a logarithmic regression was found to best approximate the

graph. The problem with using a logarithmic regression here is that it doesn't approach a limit;

this leads to it being unsuccessful in approximating the curve at the upper extreme of n, making

it of little practical use. All of these representations of the limit show that the 32 side

approximation is essentially close enough to the actual area to be credible to about the

hundredths place. This entire process need not be repeated for every twist angle and height to

radius ratio; because of the proven accuracy of the 32 side approximation, this model can be used

to find the surface area for whatever parameters are set forth.

Discussion/Conclusion:

Discovering and modeling the squeeze in the Twisted Savonius VAWT's blades as it twists

has led to the important knowledge of exactly how to build the turbine in terms of geometric

parameters. The newfound understanding of the geometric constraints of this complex shape can

be combined with the known and yet-to-be-known constraints of the materials used to build it to

create an improved turbine that is both cheaper to produce and more efficient in operation.

Restraints in twisting that extended beyond materials previously are now explained by this

theory of *geometric* squeezing of the shape. The squeeze's modeling has also simplified the

construction method, by limiting the sections of true Savonius shape to four, instead of

approaching the limit of infinity. Previously, many turbines were carefully molded into the

"true" twisted Savonius shape where *every* horizontal cross section had the same radius. This is

no longer necessary as the squeeze has been shown to be minimal, with maximum twist and

construction efficiency, when the turbine is twisted pi/4 radians, or 45° between each of the five

true cross sections, which can also act as supports.

One piece that could be overlooked is the visuals that were created using the pseudo-three-

dimensional model animations. These allow for visualization of the turbines in operation and can

be layered in *Geometry Expressions* to show what the turbines would look like on a building

from virtually any angle (street or air) in any colors, even semi-transparent; but most importantly *in operation.* Knowing the surface area of the blade also allows cost estimates to be made much more accurately. The most important surface area figure is that when there is an overall a:d (height to diameter) ratio of 2:1 (individually .5:1) and the twist is pi/4 radians for each section of each side, which is ~.82 units squared. Therefore, the total surface area for the fully twisted model at the simplest logical a:d ratio is ~3.28 units squared, a useful figure to know for efficiency and material costs purposes.

There are several limiting factors, however. The first issue is that the visual models are technically two-dimensional and depict a three dimensional shape. The use of ellipses and semi-transparent blades makes this less noticeable, but when viewing the animation of the turbine "in operation" the turbine can appear to be spinning either clockwise or counter-clockwise, depending on how you look at it, even though it is mathematically spinning clockwise. The triangle approximations are also *approximations* of the actual shape and many more triangles may be needed to better this approximation. One positive is that the angle of 45° was found by a previous study using computational fluid dynamics (Mehdi, 2006), to be the most efficient, which is now also demonstrated geometrically. Saha (2005) stated that more twist is better, which is also true in principle, and the stacking method allows higher twist angles to be used.

The data collected does allow cost estimates to be made, though exact figures will vary widely based on materials. A 2 meter tall blade (for a 3 meter turbine) could be built from the triangle approximations for under $250 with the axle/mount, but without gears, generator, and electronics. I predict that a scaled up version of the 32 side approximation could be cut out of semi-flexible fabric and stretched over the frame, then coated with something to fill any small holes, all to create a very simple, cheap and effective blade. This compares to current costs that

Figure 14: *Aerotecture's "Aeroturbine"*

can range up to $15,000-$25,000 for the *Aerotecture* "Aeroturbine", figure 14, which may be the only commercial model with a similar design (Aerotecture, 2010 [and fig. 14]), though it also has a Darrieus rotor attached. There are still many limitations to the triangle approximation models. The biggest is that because they are approximating a complex shape, more triangles could always be used to make the models. Additionally, a refined method for utilizing the triangle approximation models, without making all of the folds along each triangle, needs to be developed before the model is fully ready for use. Overall, however, the cheaper cost and simpler construction open the door for widespread use of the Twisted Savonius design methods in rooftop settings. With this simpler design and construction method, they could feasibly and inexpensively be placed on all suitable buildings to generate large amounts of power. And now that the geometrical constrains of the Twisted Savonius shape have been determined, additional research on the design will be able to focus on finer details, like materials and methods.